Introduction to Phase Encoding in MRI

By
Henrik BW Larsson

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Phaseencoding
In order to encode the image or the imaged object along the so-call phase encoding direction, here the y direction (but the direction could also be in z or x direction), a gradient along the y direction is applied, and thereafter the signal is sampled. In fact it is necessary to apply this gradient several times, each time increasing the gradient by an equidistant amount. In a MRI sequence diagram this procedure is indicated by the phase encoding table, see Figure 1. As seen this table consist of 32 steps in this example, each lasting for 0.250 ms, and with an increment of 0.734085 mT/m (milli Tesla pr metre). One could go through this table from the bottom to the top, or from the top to the bottom, this is called linear phase encoding. One could also go from the middle and outward like 0, 1, -1, 2, -2 etc, this is called low-high phase encoding. Note that one particulate step corresponds to applying no gradient at all, and for the low-high encoding, this would then be the first step.

Dimension
The dimension of the increment of the phase encoding table, and the number of steps are very important and related to the field of view in the y direction (FOV_y) and the resolution of the image along the y-axis. This is the topic in the following. As a starting point the number of phase encoding steps correspond the number of pixels along the y-axis, unless parallel imaging is used. We will start at looking at one unit step of the phase encoding table, and this is here called ΔG_y. Such a step is seen in figure 2 marked with red. The table has been change from 32 steps to 16 steps in order to enhance visual clarity in the following.

Let us first recapitalize one of the most important equations in MRI, that is the Lamor equation:

$$\omega = \gamma B_0 \qquad [1]$$

If also a gradient along the y-axis is applied, then the equation becomes:

$$\omega = \gamma(B_0 + G_y y) \quad [2]$$

We can remove the B_0 term, and this correspond to a representation in the rotating frame. So we are left with:

$$\omega = \gamma G_y y \quad [3]$$

If the G_y gradient is on for a time period of ΔT_y, then this gradient introduce a phase shift which is:

$$\alpha = \omega \Delta T_y = \gamma G_y y \Delta T_y \quad [4]$$

It can immediately be seen that the induced phase shift is a function of the position along the y-axis. As stated above, after each step of the phase encoding gradient, the MR signal is acquired. Now

recall the Nyquist criteria, which state that in order to represent a digitalized signal correctly, it must be sampled twice the highest frequency encountered in the signal, that is, it should be sampled two times per cycle. One gradient step is called ΔG_y here and the duration is called ΔT_y, see Figure 2. We find the largest phase shift at the top (or the bottom) of the FOV, so the condition to be satisfied is:

$$\alpha = \pi = \gamma \Delta G_y \frac{FOV_y}{2} \Delta T_y \Leftrightarrow 2\pi = \gamma \Delta G_y FOV_y \Delta T_y \quad [5]$$

The value of the gyromagnetic ratio is:

$$\gamma = 2.675 \text{ x } 10^8 \text{ rad s}^{-1} \text{ T}^{-1}$$
$$\gamma' = \frac{\gamma}{2\pi} = 42.57 \text{ MHz T}^{-1} \quad [6]$$

If we use the last expression for the gyromagnetic ratio and insert it into Eq.[5], we get:

$$1 = \gamma' \Delta G_y \, FOV_y \, \Delta T_y \quad [7]$$

If FOV_y is chosen to 128 mm, and the time duration of the gradient is set to 0.250 ms, then we can calculate the necessary gradient to:

$$\Delta G_y = \frac{1}{\gamma' FOV_y \, \Delta T_y} = 0.734085 \text{ mT/m} \quad [8]$$

Equation [7] can also be expressed as:

$$FOV_y = \frac{1}{\gamma' \Delta G_y \, \Delta T_y} \equiv \frac{1}{\Delta k_y} \quad [9]$$

Note the definition in the last equation, and that the dimension of Δk_y is mm^{-1}, which indicate spatial frequency.

The phase encoding table

So what is the effect of one phase encoding step ?. This is seen from Figure 3. Note that the spin group at position y = 0 do not change the phase, but the spin group in the top pixel obtain a maximum phase change of exactly 180 degree as required. After this phase encoding the signal is sampled. In fact the MR signal is in proportion to the sum of all the spin groups added together as vectors. The effect of the next phase encoding step is seen in Figure 4. Note that the phase of the top-pixel again changes by additional180 degrees. So the pixel experience the greatest phase shift will be sampled exactly twice during one cycles. Also note that the pixel in the middle does not change phase in any situation. Figure 5 shows what happens if we use the maximal phase encoding step, and Figure 6 shows what happens if we use the centre phase encoding step; i.e. no gradient is applied. In this case no phase change is introduced at all along the y-axis.

The situation just explained is a little odd in that in order to observe this phase change, we should use a phantom, which just have a y dimension of the FOV_y and should have equal magnetisation. I practice some of the positions, or pixels, just contain air, and will not contribute with a signal and the phase is not defined. And of course the signal from the pixels may be weighted by T_1 and T_2 or T_2*. However, this does not matter, as the important point to make is that every pixel along the y-axis (if they exist) has unique change of phases depending on the exact position along the y-axis as we proceed along the phase encoding table. This can also be interpreted as different frequency along the y-axis. Take for instance the top pixel, this will experience a phase change of 180 degree every time it is measured. The measuring time interval can be interpreted as being 0.250 ms so the top pixel has in a sense a frequency of π / 0.250 ms^{-1} = 2000 Hz. So people call this kind of frequency a pseudo frequency. The pixel just below the top pixel will in this sense have a frequency of 7 π / 8 /0.250 ms^{-1} = 1750 Hz, and so on. The center pixel will have a frequency of 0 Hz. In this sense every position along the y-axis is characterised by one unique frequency. By the way, what is the sign of the frequency of the pixels for negative y positions?

Aliasing in the phase encoding direction

Let us exam what happens if we have material outside the FOV_y , keeping a nominal FOV_y = 128 mm, and that the phase encoding table is the same as before. Figure 7 shows what happens when a phase encoding step as seen in Figure 2, is applied. The obtain phase shift of possible spin group just outside the FOV_y on the top (marked with green arrow), behaves exactly as the spin group positioned at the bottom of the FOV (also marked with green). Likewise, the two spin groups marked with orange will obtain exactly the same phase shift. This will occur for all phase encoding steps, and just as an example the result of the next phase encoding step is seen in Figure 8. We still only have 16 pixels, so spin groups outside the FOV will be aliased, or folded back inside the FOV and placed corresponding to the spin group with a similar phase behaviour, because these spin groups cannot be differentiated by any means. This should give a strong hind of why the nose can be seen at the back of the head, when having the phase encoding direction in the anterior – posterior direction, investigating large headed people. And of course, the back of the head could be aliased to the position of the nose.

Increasing the FOV_y

In order to avoid aliasing as described above, the FOV_y has to be increased. So we will increase the FOVy from 128 mm to 160 mm. Again we have to satisfy Eq.[7]. If we set the time duration ΔT_y = 0.250 ms, as before, we can calculate the necessary phase encoding gradient step from Eq.[8] :

$$\Delta G_y = \frac{1}{\gamma' FOV_y \Delta T_y} = 0.587268 \text{ mT/m}$$

As seen the value of the gradient step has to be decreased in order to avoid aliasing. The situation is shown in Figure 9 for the first phase encoding step. So the conclusion to be made is that in order to increase field of view, each step in the phase encoding table has to be smaller, when keeping the number of phase encoding step constant, in this case 16 steps. This also has as a consequence that the spatial resolution decreases (the length of the single pixel increases in the y direction), as the resolution in the y direction was previously 128 mm/16 = 8 mm, and increases to 160 mm/16 = 10 mm.

Increasing the spatial resolution in the phase encoding direction

The spatial resolution in the phase encoding direction is given by.

$$\Delta y = \frac{FOV_y}{n_y} \quad [10]$$

where n_y is the number of phase encoding steps. So if the spatial resolution is to be increased by a factor of 2, meaning that Δy is decreased by a factor of 2, then the number of phase encoding steps has to be doubled. If the FOV_y = 128 mm and ΔT_y = 0.250 ms as before, the gradient step to be used is given by Eq.8. As neither the FOV_y nor the ΔT_y has been changed we still get ΔG_y = 0.734085 mT/m. The situation is shown in Figure 10. The conclusion to be made is that the increment of the phase encoding step is kept constant, but we need more steps. This is equivalent by stating that we acquire information at higher spatial frequencies or at higher k_y-space values.

Increasing both FOV_y the spatial resolution in the phase encoding direction

Now we want to combine the two situations, increasing the FOV_y to 160 mm and have a spatial resolution of Δy = 4 mm.

Accordingly, the number of phase encoding steps are 160 mm / 4mm = 40. So n_y = 40. Using Eq.[8] again, we get:

$$\Delta G_y = \frac{1}{\gamma' FOV_y \Delta T_y} = 0.587268 \text{ mT/m}$$

This situation is shown in Figure 11.

Try to estimate how long time it takes to acquire an image for the various presented situations, assuming that one phase encoding step takes one TR.

The k_y space interpretation
Comming later

Closing remarks
In this presentation we have ignored non-ideal gradient profiles, meaning that an instantaneous increase of a gradient is hard to achieve, and normally the raise time profile has to be taken into account. However, the principally findings are unchanged.

Figure 1.

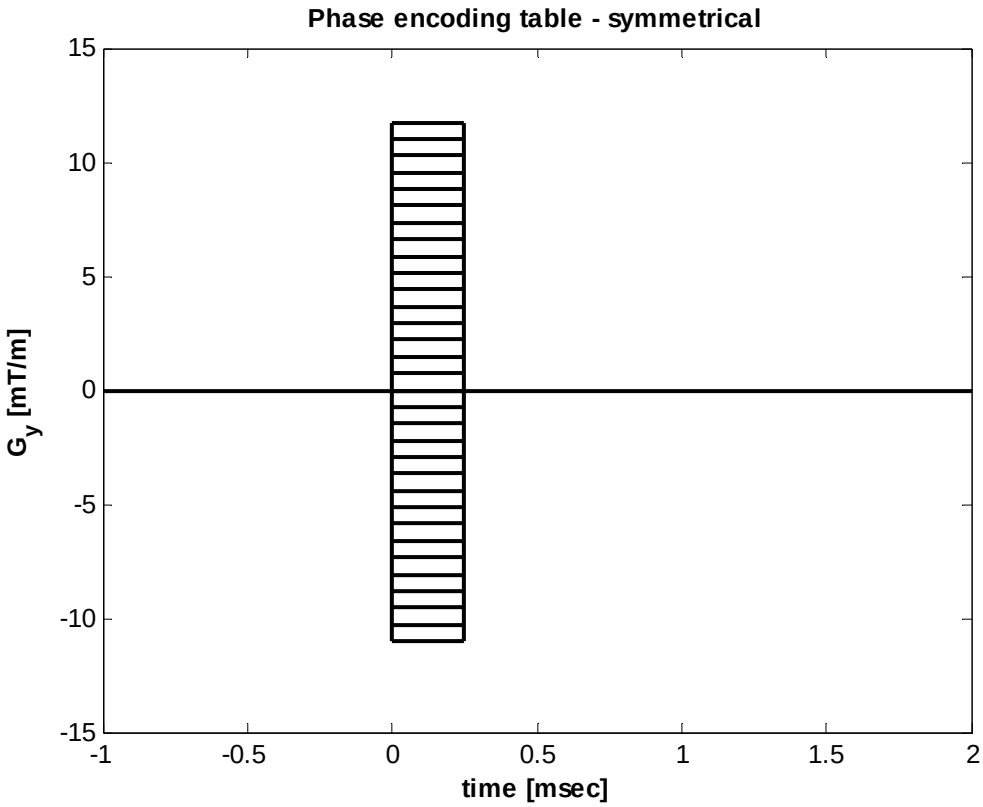

Phase encoding table - symmetrical
15
10
5
0
-5
-10
-15
-1
-0.5
0
0.5
1
1.5
2
G_y [mT/m]
time [msec]

Figure 2.

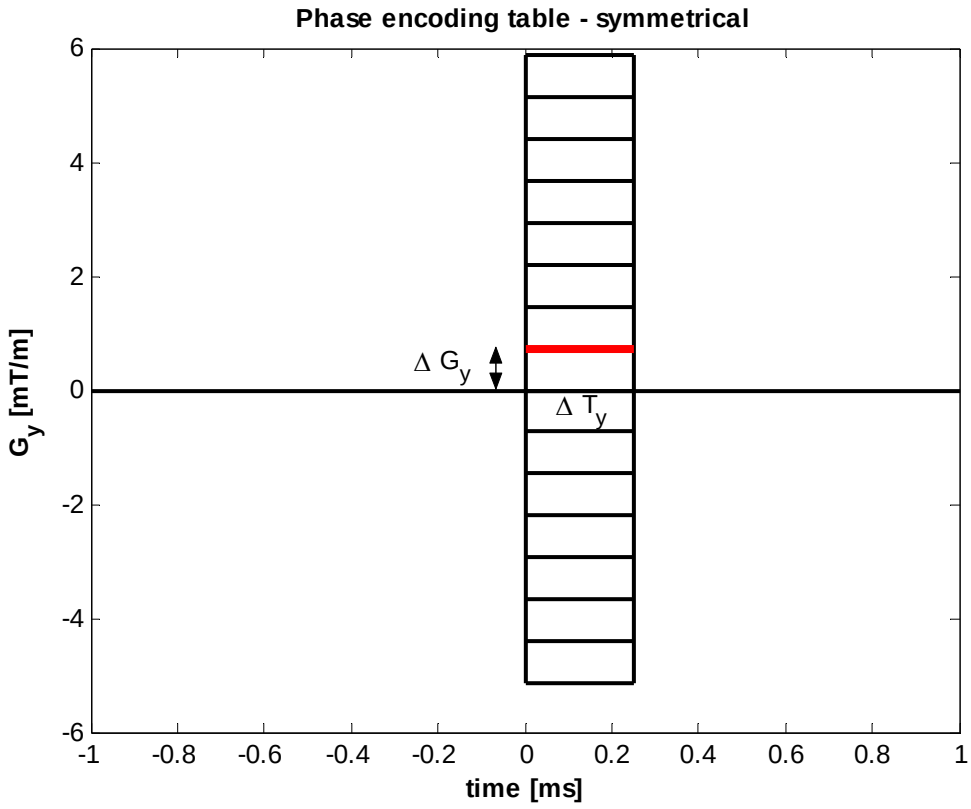
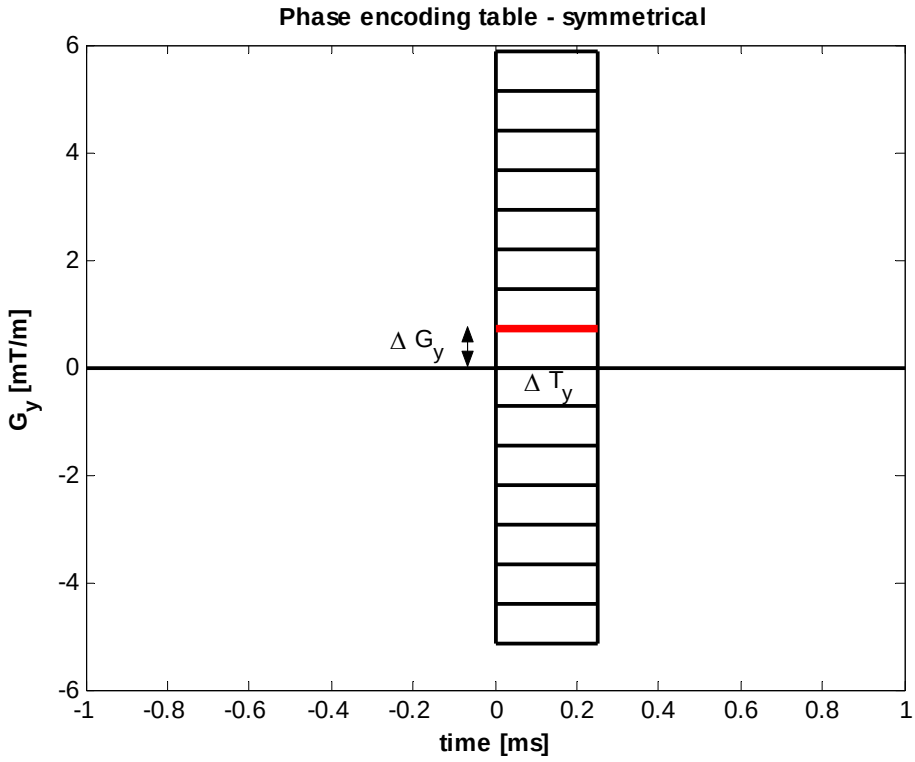

Phase encoding table - symmetrical
6
4
2
0
-2
-4
-6
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
G_y [mT/m]
time [ms]
Δ G_y
Δ T_y

Figure 3.


Phase encoding table - symmetrical
Δ Gy
Δ Ty
Gy [mT/m]
time [ms]
6
4
2
0
-2
-4
-6
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

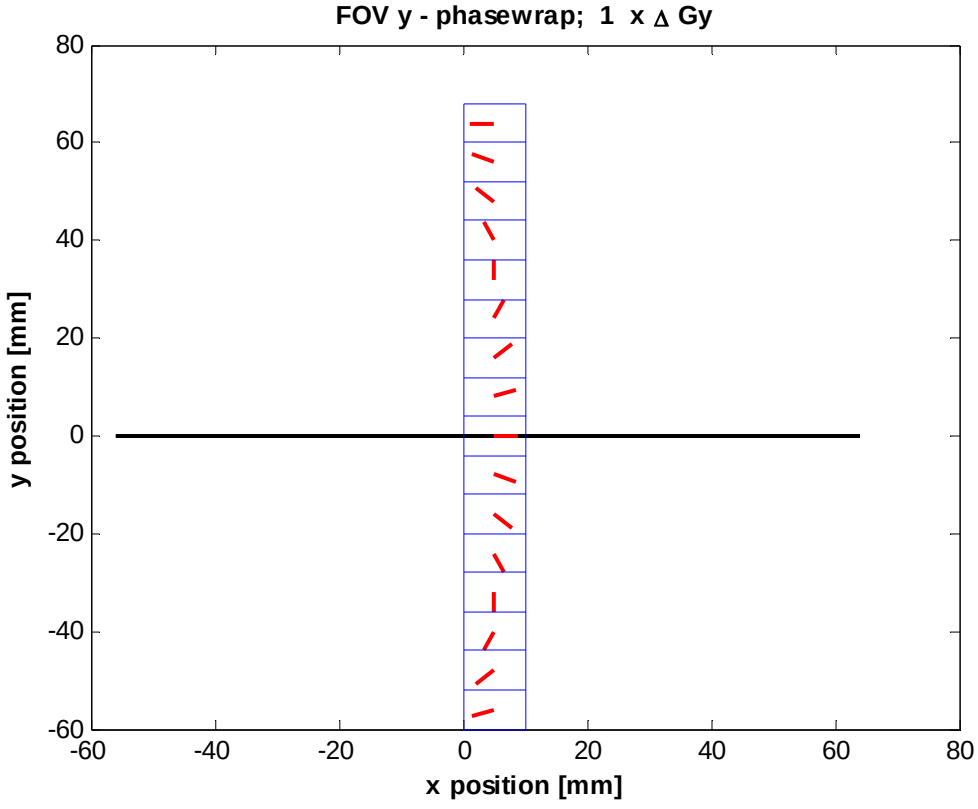
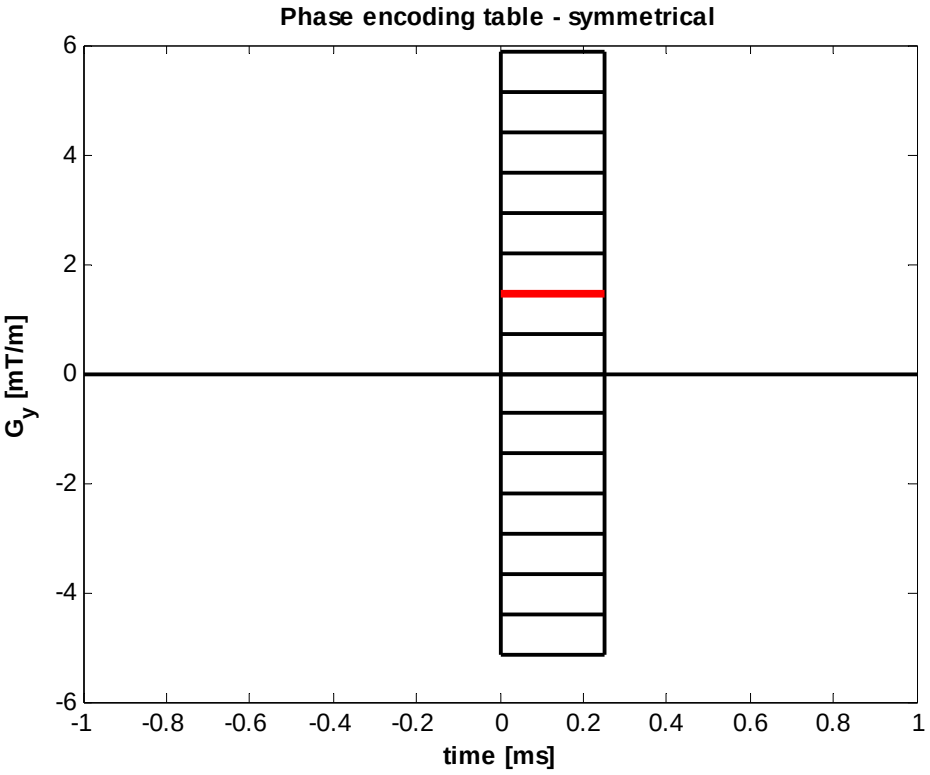

FOV y - phasewrap; 1 x Δ Gy
y position [mm]
x position [mm]
80
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60
80

Figure 4.


Phase encoding table - symmetrical
G_y [mT/m]
time [ms]
6
4
2
0
-2
-4
-6
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

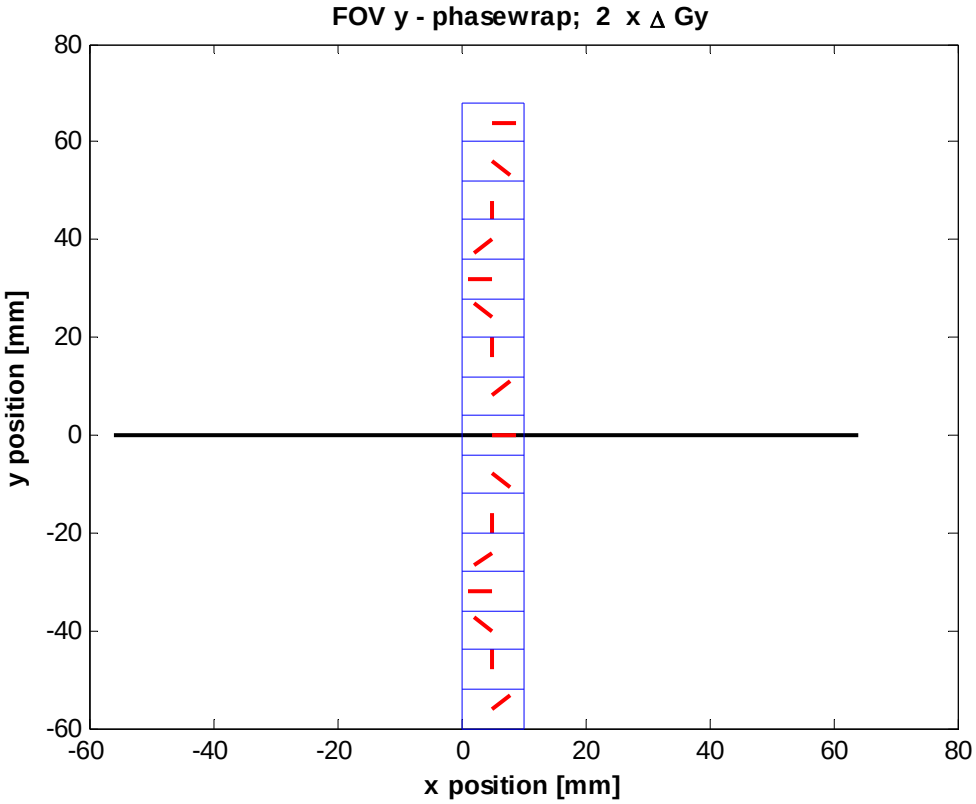

FOV y - phasewrap; 2 x Δ Gy
y position [mm]
x position [mm]
80
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60
80

Figure 5.

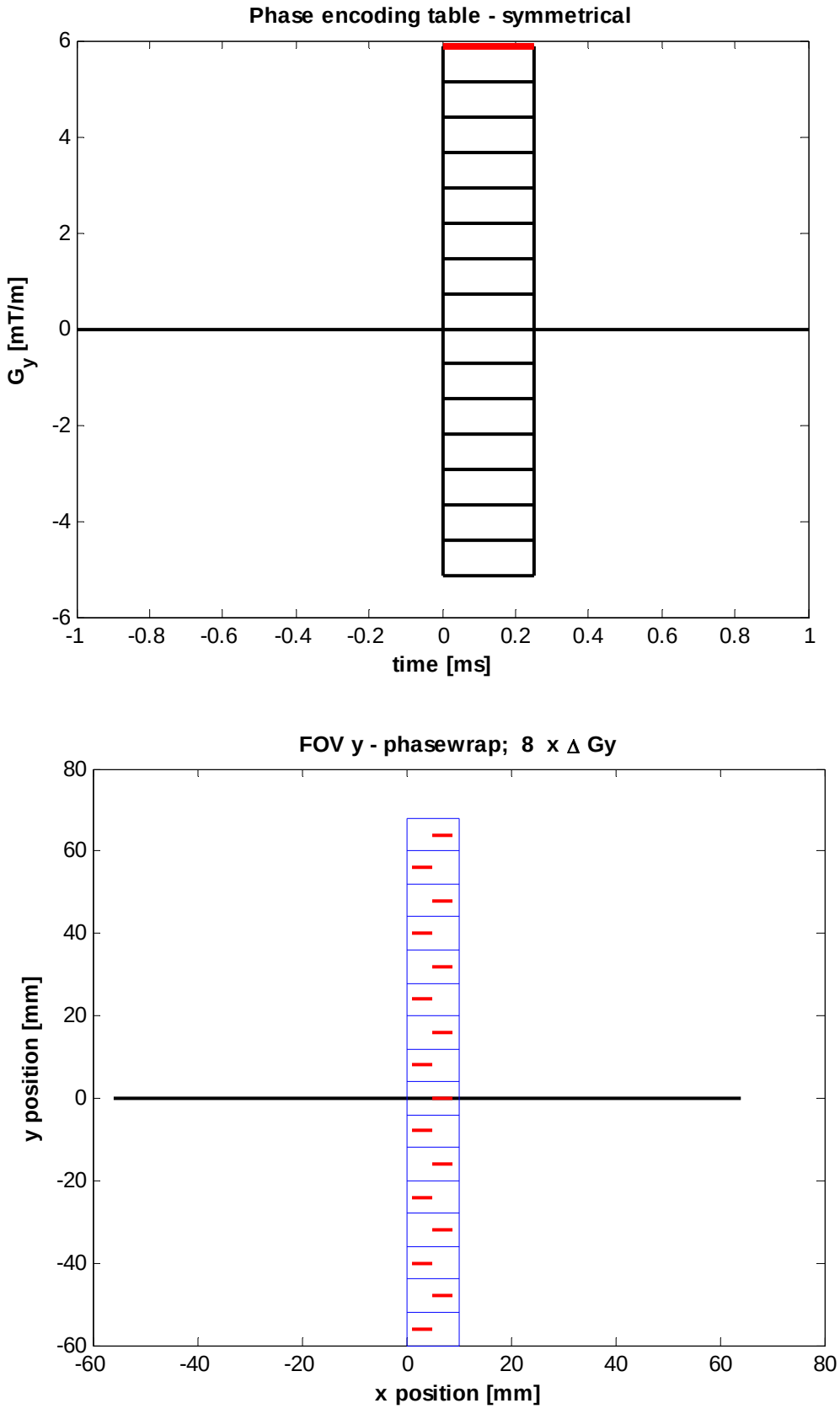
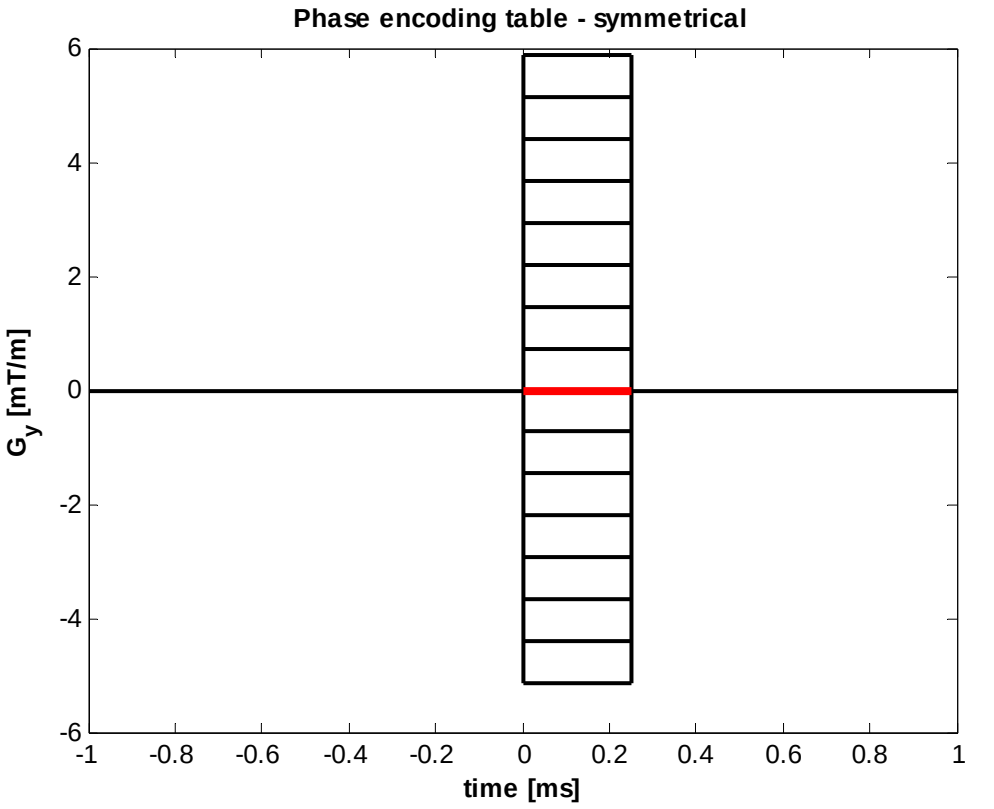

Phase encoding table - symmetrical
G_y [mT/m]
time [ms]
FOV y - phasewrap; 8 x Δ Gy
y position [mm]
x position [mm]

Figure 6.


Phase encoding table - symmetrical
6
4
2
0
-2
-4
-6
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
G_y [mT/m]
time [ms]

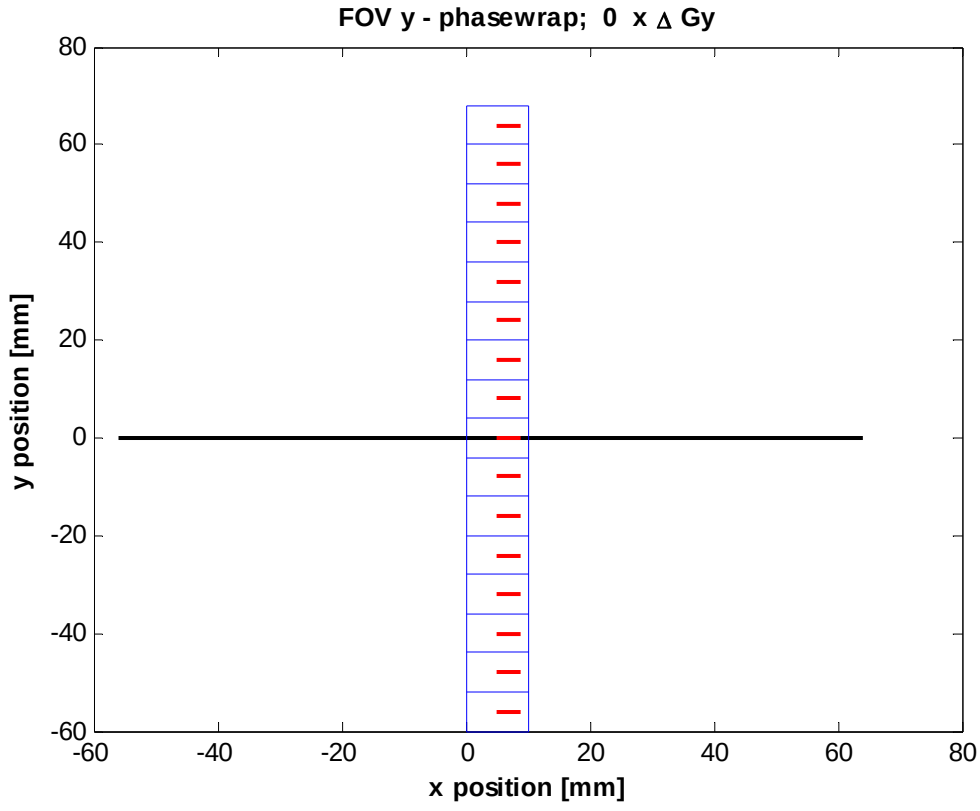

FOV y - phasewrap; 0 x Δ Gy
80
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60
80
y position [mm]
x position [mm]

Figure 7.

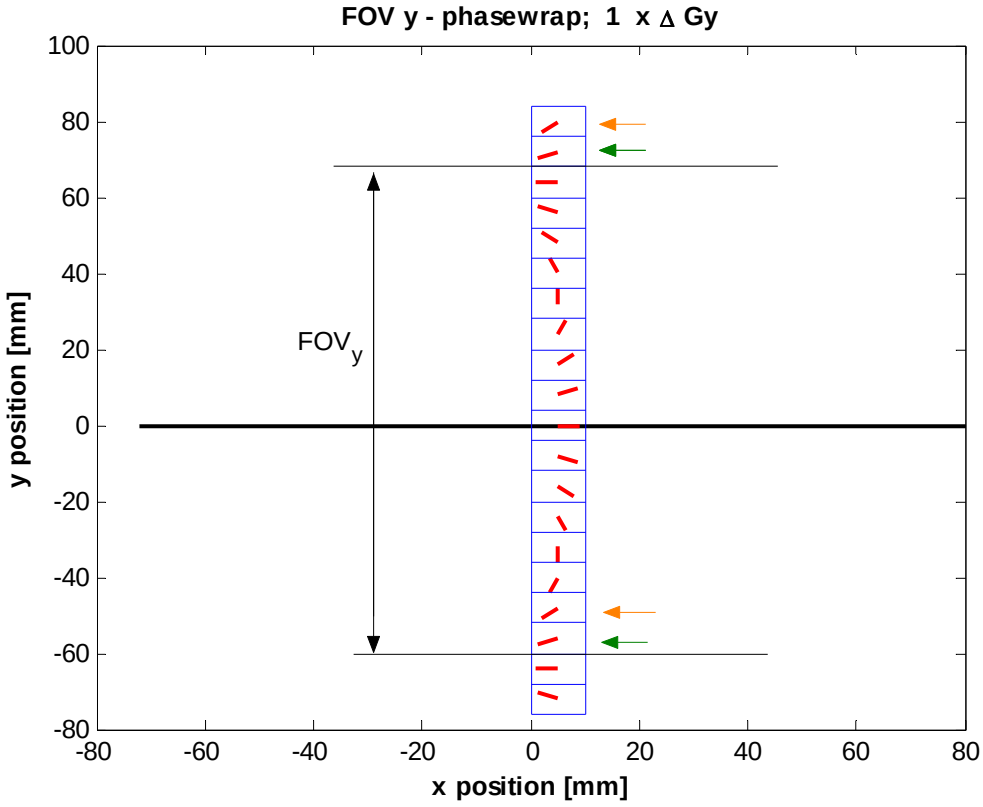

FOV y - phasewrap; 1 x Δ Gy
y position [mm]
x position [mm]
FOV$_y$
100
80
60
40
20
0
-20
-40
-60
-80
-80
-60
-40
-20
0
20
40
60
80

Figure 8.

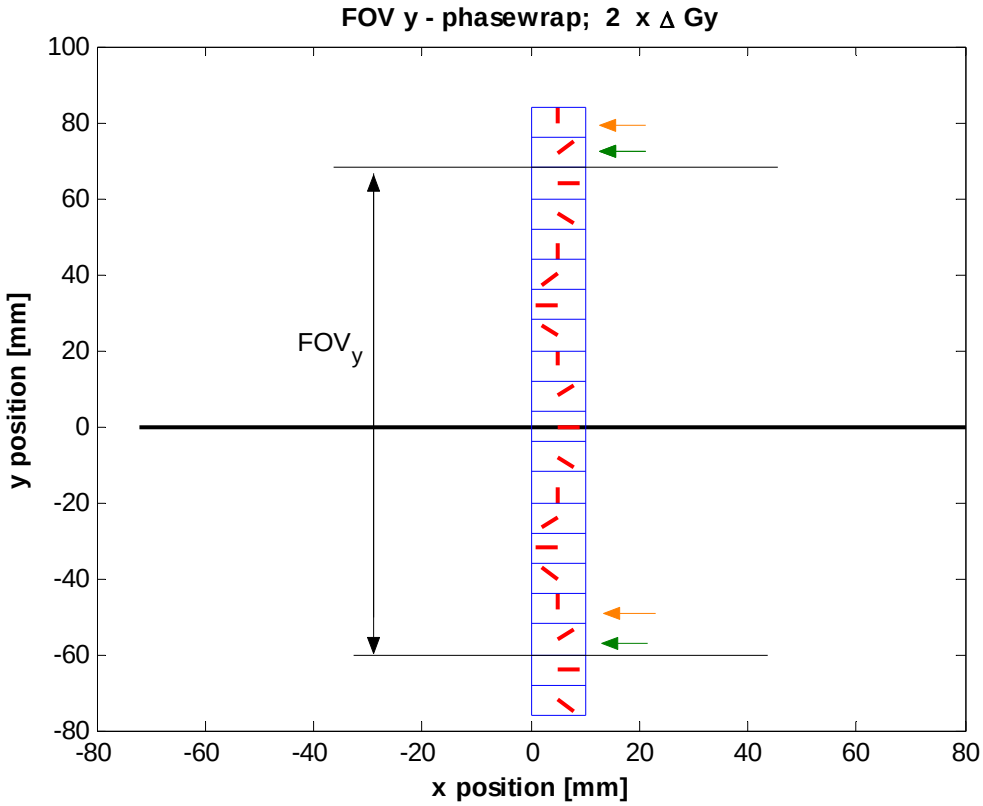

FOV y - phasewrap; 2 x Δ Gy
y position [mm]
x position [mm]
FOV$_y$
100
80
60
40
20
0
-20
-40
-60
-80
-80
-60
-40
-20
0
20
40
60
80

Figure 9.

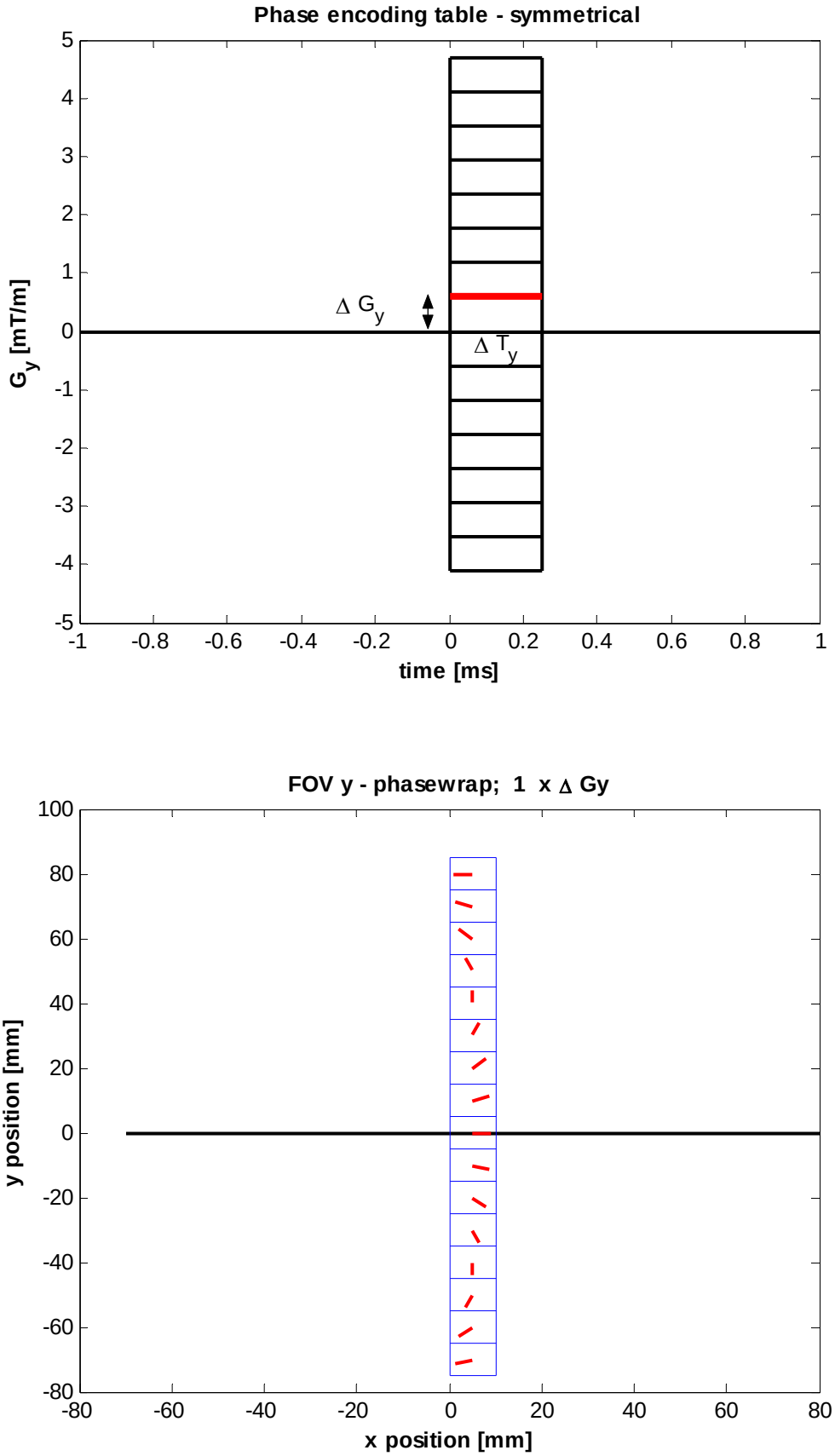
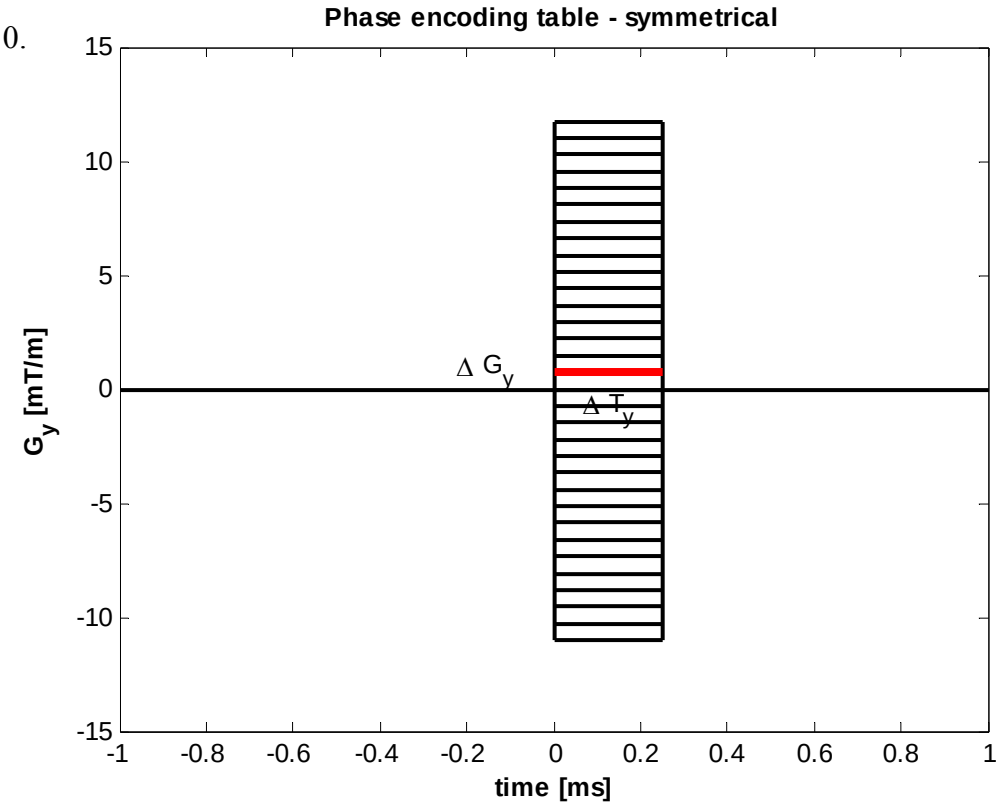

Phase encoding table - symmetrical
Δ Gy
Δ Ty
Gy [mT/m]
time [ms]
FOV y - phasewrap; 1 x Δ Gy
y position [mm]
x position [mm]

Figure 10.


Phase encoding table - symmetrical
15
10
5
0
-5
-10
-15
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
G_y [mT/m]
time [ms]
Δ G_y
Δ T_y

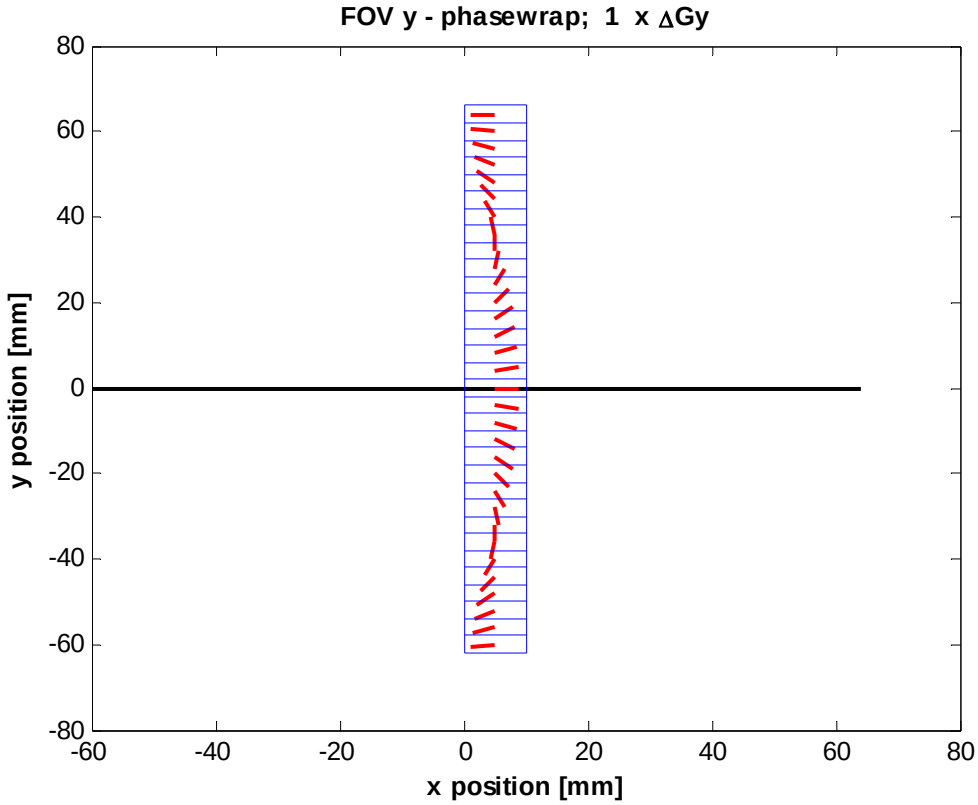
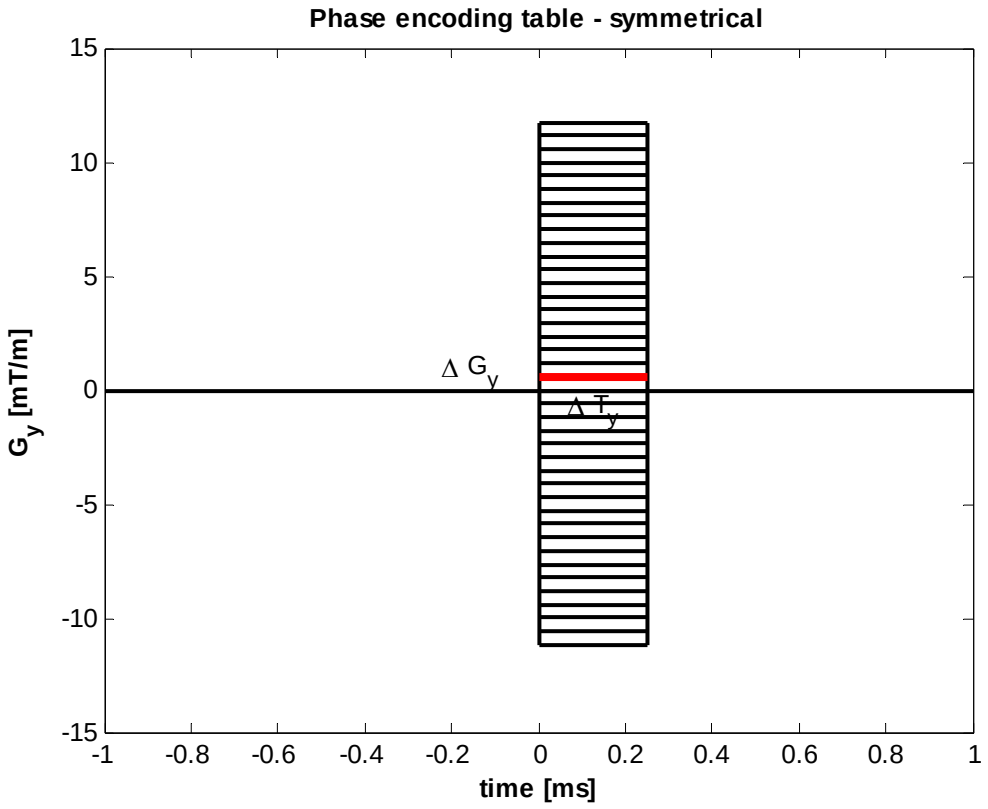
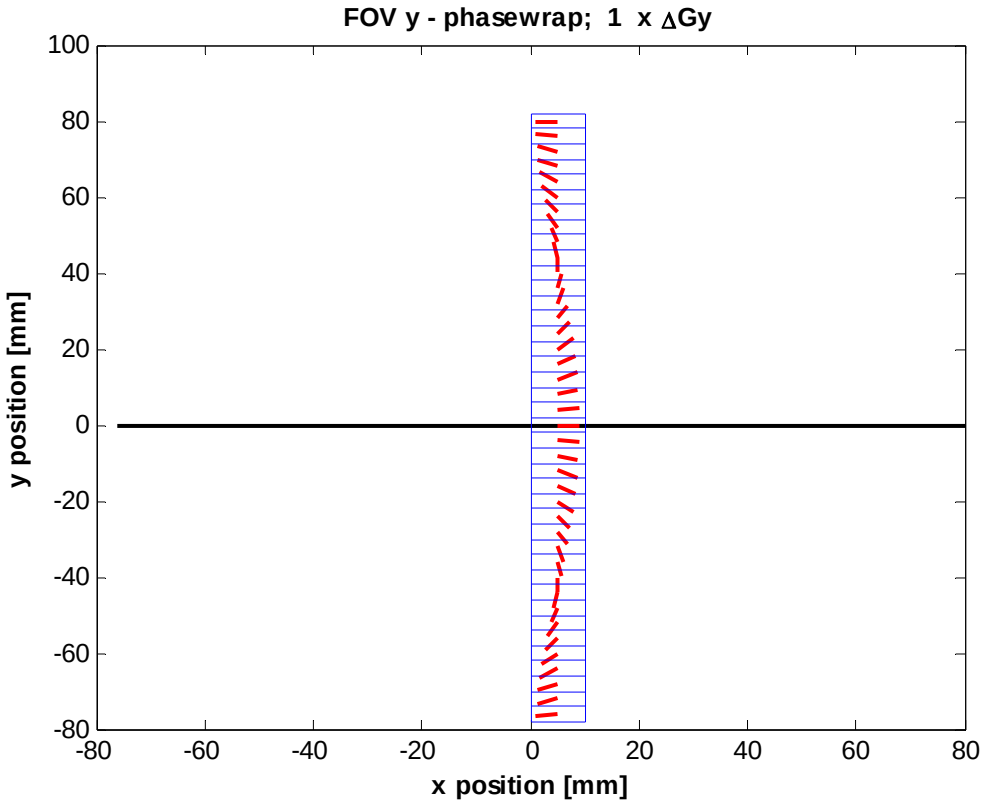

FOV y - phasewrap; 1 x ΔGy
80
60
40
20
0
-20
-40
-60
-80
-60
-40
-20
0
20
40
60
80
y position [mm]
x position [mm]

Figure 11.


Phase encoding table - symmetrical
15
10
5
0
-5
-10
-15
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
G_y [mT/m]
time [ms]
Δ G_y
Δ T_y


FOV y - phasewrap; 1 x ΔGy
100
80
60
40
20
0
-20
-40
-60
-80
-80
-60
-40
-20
0
20
40
60
80
y position [mm]
x position [mm]